Properties of DTFS

(1) Linearity

$x(n) \longleftrightarrow X_k$

$y(n) \longleftrightarrow Y_k$

Let Two periodic signals with period N,

$z(n) = a x(n) + b y(n)$

$\longleftrightarrow Z_k = a X_k + b Y_k$

$$x(n) = \sum_{k=N} X_k e^{jk\omega_0 n}$$

$$X(k) = \frac{1}{N} \sum_{n=N} x(n) e^{-jk\omega_0 n}$$

(DFT.)

$$Z_k = \underbrace{\frac{1}{N} \sum_{n=N} x(n) e^{-jk\omega_0 n}} = \frac{1}{N} \underbrace{\sum_{n=N} [a x(n) + b y(n)] e^{-jk\omega_0 n}}$$

$$Z_k = a X_k + b Y_k.$$

(2) Time Shifting:

$x(n) \longleftrightarrow X_k$.

$y(n) = x(n-n_0) \longleftrightarrow Y_k = X_k e^{-j\omega_0 n_0}$

The magnitude of its fourier series coefficient remain altered. $|Y_k| = |X_k|$

$$Y_k = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-jk\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x(n-n_0) e^{-jk\omega_0 n}.$$

if $m = n - n_0$, $n \to 0$, $m = -n_0$.

$m = N-1-n_0$ as $n \to (N-1)$.

$$Y_k = \frac{1}{N} \sum_{m=-n_0}^{N-1-n_0} x(m) e^{-jk\omega_0 (m+n_0)}$$

$$= \frac{1}{N} \sum_{m=-n_0}^{N-1-n_0} x(m) e^{-jk\omega_0 m} e^{-jk\omega_0 n_0}$$

$$Y(k) = X(k) e^{-jk\omega_0 n_0}$$

5(8)

③ **Frequency Shifting**:

X_{k+m}

$$x(n) \longleftrightarrow X_k$$

$$y(n) = e^{jM\omega_0 n} x(n) \longleftrightarrow Y_k = X_{k-m}$$

By def. $Y_k = \frac{1}{N} \sum_{n=N} y(n) e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-jk\omega_0 n}$

$$= \frac{1}{N} \sum_{n=0}^{N-1} e^{jm\omega_0 n} \cdot x(n) e^{-jk\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j(k-M)\omega_0 n} = X_{k-m}$$

④ **~~Time~~ Reversal**: $x(n) \longleftrightarrow X(k)$

$$y(n) = x(-n) \longleftrightarrow Y_k = X_{-k}$$

The time reversal applied to a discrete time signal results in a time reversal of the corresponding sequence of F.S. coef

$$Y_k = \frac{1}{N} \sum_{n=N} y(n) e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-jk\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x(-n) e^{-jk\omega_0 n} = \frac{1}{N} \sum_{m=-(N-1)}^{0} x(m) e^{-j(-k)\omega} = X_{-k}$$

if $x(n)$ even = F.S. even

$$x(-n) = x(n)$$

$$X_{-k} = X_k$$

if $x(-n) = -x(n)$ odd

$$X_{-k} = -X_k.$$

5) Time Scaling: → $t \propto \frac{1}{f}$ . . 5(9)

Let m be a positive integer

$$x_{(m)}(n) = \begin{cases} x(n/m), & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$$

$x_m(n)$ can be obtained from $x(n)$ by placing $(m-1)$ zeroes betn. successive value of original signal.

$x\, m/n = m X_K$.

$x(n) \leftrightarrow X_K$.

$$\boxed{y(n) = x_m(n) \leftrightarrow Y_K = \frac{1}{m} X_K}$$

Proof:

$$Y_K = \frac{1}{mN} \sum_{n=0}^{m(N-1)} y(n)\, e^{-jK(\omega_0/m)n}$$

$$= \frac{1}{mN} \sum_{n=0}^{m(N-1)} x(m)(n)\, e^{-jK(\omega_0/m)n}$$

$$= \frac{1}{mN} \sum_{n=0}^{m(N-1)} x\left(\frac{n}{m}\right) e^{-jK(\omega_0/m)n}$$

Let $r = \frac{n}{m}$, if $r = 0$ as $n = 0$

if $r = \frac{m(N-1)}{m} = N-1$

$$\boxed{Y_K = \frac{1}{m} \cdot \frac{1}{N} \sum_{r=0}^{N-1} x(r)\, e^{-jk\omega_0 r} = \frac{1}{m} X_K}$$

(6) Periodic Convolution:

$x(n) \leftrightarrow X_K$,

$y(n) \leftrightarrow Y_K$

then $z(n) = x(n) \circledast y(n)$

$$= \sum_{r=N} x(r)\, y(n-r) \leftrightarrow Z_K = N X_K Y_K$$